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Parameter-induced logical stochastic resonance

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ABSTRACT

In this paper, we discuss how to get adaptive logical stochastic resonance (LSR) by modulating the parameters of nonlinear system. The effects of linear and nonlinear coefficients of a quartic-bistable system on the system's response to feeble input signals in noisy background are investigated. Genetic Algorithm is applied to search for the optimal system parameters in the given noise. The success probability of obtaining desired logic output is used as the fitness function. Experimental results show that the system can achieve robust logic operation in a wide range of noise intensity by adjusting the parameters. The study might provide an example of the application of parameter-induced LSR in engineering practice.

1. Introduction

Benzi et al. [1–3] firstly put forward the concept of stochastic resonance (SR), which is brought up to address the problem of the apparent synchrony between glacial periods and the variations of solar energy influx. Since then, SR has continuously attracted considerable attention from various fields and it has found many applications in electric circuits, lasers, chemical systems, etc. Gammaitoni et al. [4] have written an extensive review for the classical SR theory and its important applications. In particular, several novel types of stochastic resonance which are especially relevant to neuro-computation are discovered recently, to name but a few, delayed-induced multi-resonance [5,6], coherence resonance [7–9], pacemaker induced stochastic resonance in neuronal systems [10–13], etc.

Recently, Murali et al. [14] introduced the concept of logical stochastic resonance (LSR). They investigated the response of a simple threshold detector to input signals, consisting of two random square waves, and showed that the interplay of nonlinearity and moderate density of noise can yield logic behavior (NOR and NAND). Another research of Murali et al. [15] demonstrated LSR via a circuit implementation using a linear resistor, a linear capacitor and four CMOS-transistors.

LSR suggests a new way of implementing reconfigurable and reliable logic gates in the presence of noise. Although LSR is a recent idea, the number of studies on LSR is growing fast. For instance, Zhang et al.[16,17] investigated the effects of OU noise and $1/f$ noise on LSR. Animesh et al. [18] found that dynamical behavior equivalent to LSR can also be obtained without noise. Remo et al. [19] extended the study of LSR from the bistable system to the multistable (tri-stable) system given by piecewise functions and obtained XOR logic. Zhang et al. [20] proposed that the LSR phenomenon in a class of 3-well system can be successfully induced by additive or multiplicative Gaussian colored noise, and obtained the approximate Fokker–Plank equation by using decoupling approximation. Wang et al. [21,22] investigated LSR phenomenon in the presence of stable distribution noise and realized Set–Reset latch operation in a bistable system driven by Ornstein–Uhlenbeck noise. Studies about LSR in chemical, nanomechanical, optical and biological systems have been investigated by Sinha et al. [23], Guerra et al. [24], Zamora-Munt et al. [25] and Ando et al. [26].

However, all the LSR phenomena are gained by modulating noise density in the above studies. Although adding noise to get LSR is a useful method in the context of "under-resonant", wherein the background noise strength is lower than needed, this method is not appropriate under "over-resonant" circumstance, wherein the background noise is bigger than needed and cannot be filtered. Xu et al. proposed that tuning the parameters of the bistable system can achieve conventional stochastic resonance and pointed out that although increasing noise intensity will enhance the system response speed, the output signal-to-noise ratio is degraded [27]. At the same time, according to the information theory, at high level of noise, the amount of information in the output signal will decrease with the addition of more noise [28]. So in order to get best LSR, tuning system parameters are a better choice than modulating the background noise.

In this paper, we propose achieving LSR by adjusting the system parameters and apply Genetic Algorithm to search for the optimal system parameters. The outline of the paper is organized as follows: the quartic-bistable system used in this paper is depicted in Section 2, Section 3 studies the effects of the linear and nonlinear coefficients, Section 4 proposes self-adaptive

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Table 1
Relationship between two logic inputs and the logic output of the four fundamental AND, NAND, OR, and NOR gates. The combination of the four fundamental logic gates can be used to construct any logic circuit.

Input set (I_1,I_2)	OR	NOR	AND	NAND
(0,0)	0	1	0	1
(0,1)/(1,0)	1	0	0	1
(1,1)	1	0	1	0

LSR based on Genetic Algorithm. We finish with final remarks and conclusions in Section 5.

2. Bistable system based LSR

Without loss of generality, considering an overdamped Brownian particle in a double well potential driven by Gaussian white noise, the Langevin-type equation is given by

$$\dot{x} = -\dot{U}(x) + r + I(t) + \xi(t) \quad (1)$$

where $U(x)$ denotes the reflection-symmetric quartic potential

$$U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4 \quad (2)$$

where a and b are the linear and nonlinear coefficients of potential, r denotes the bias constant, and $I(t)$ denotes the low amplitude input signal consisting of $I_1(t)$ and $I_2(t)$, with $I_1(t)$ and $I_2(t)$ being two-value signals. To facilitate subsequent treatment, we set $I_1(t)$ and $I_2(t)$ to be logical 0 when its value is -0.5, and logical 1 when its value is 0.5. $\xi(t)$ is a zero-mean, Gaussian white noise with autocorrelation function $\langle \xi(t)\xi(0) \rangle = 2D\delta(t)$, with D being the strength of the noise.

In system (2), there are two potential wells at $x_+ > 0$ and $x_- < 0$. For logic OR, we define the output to be logical 1 when it is in the well at x_+, and logical 0 when it is in the other well at x_-. Similarly, we can get AND, NAND and NOR by defining the two wells' different logical meanings. The logical operation of the system can be checked based on the truth table (Table 1).

3. Effects of parameters on LSR

Firstly, we study the effects of $a$and b on the responses of LSR in the bistable system. The outcomes of Eq. (1) with three different sets of a and b are displayed in Fig. 1. According to potential well depth given by $\Delta U = a^2/4b$, the corresponding potential barrier height for the three different sets is 5, 0.05, and 0.42. It can be seen that for the given value of bias r and noise intensity D, the set of $a=0.5$ and $b=0.15$ brings in a clear OR logical operation (Fig. 1(d)). Fig. 1(b) shows "under-resonant" circumstance, where the particle lacks adequate power to jump to the right well in time, and Fig. 1(c) indicates "over-resonant" circumstance, where the particle hops arbitrarily between the two wells.

To be more explicit, we calculate the probability of getting the right logic output P to indicate the consistency of obtaining a given logic operation. We quantify P as follows: use the random permutated combination of 4 logical inputs sets (0, 0), (0, 1), (1, 0) and (1, 1) to drive the system over some reasonable time τ for N runs, with each input set driving for $\tau/4$. In each run, sample $x(t)$ for the four input sets we calculate the correct probability P_i, the ratio of correct logical output number of sampled $x(t)$ to the total number of sampled $x(t)$. In order to reduce the effect of relaxation time, we sample $x(t)$ in different input sets with a reasonable time delay. In this strategy, the probability P is given by

$$P = \frac{\sum_{i=1}^{N} p_i}{N} \quad (3)$$

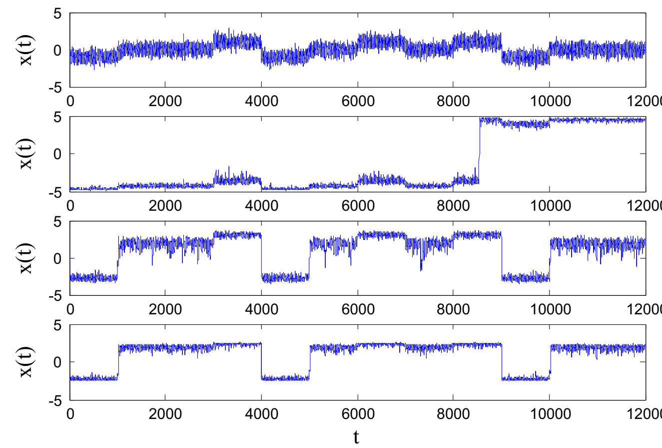

Fig. 1. From top to bottom, (a) stream of input $I_1(t)+I_2(t)+\xi(t)$, panels (b)–(d) show the outcome of the system with (b) $a=1$, $b=0.05$, (c) $a=0.1$, $b=0.05$ and (d) $a=0.5$, $b=0.15$. Here, $r=0.2$, $D=0.5$. It can be observed that clear OR logic gate operation is obtained in (d).

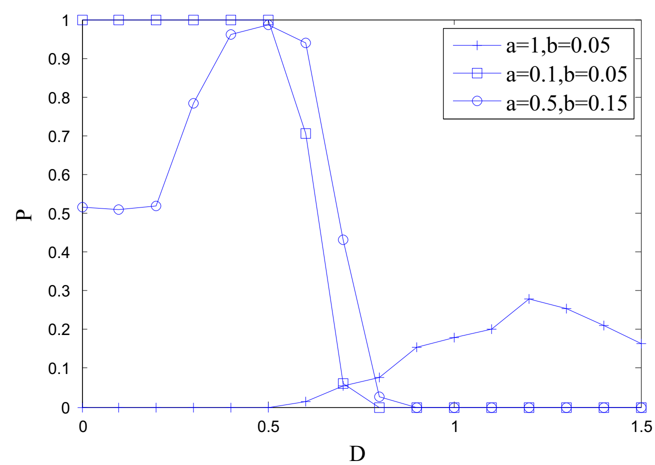

Fig. 2. In OR logic operation, the success probability P versus noise intensity D is shown with the bias $r=0.2$. The corresponding potential barrier height for the three different sets is 5, 0.05, and 0.42, respectively for ($a=1$, $b=0.05$), ($a=0.1$, $b=0.05$) and ($a=0.5$, $b=0.15$).

The success probability P as a function of noise intensity D is plotted at three different sets of a and b, when $r=0.2$, in Fig. 2. It is clear that when the potential barrier is very low ($a=0.1$ and $b=0.05$), the system is sensitive to noise. It can achieve 100% accuracy when the noise density is small, but once the noise is strong the correct probability decreases to 0 quickly. On the other hand, when the potential barrier is tremendously high ($a=1$ and $b=0.05$), although the correct probability is almost zero when the noise density is small, the system behaves better at the strong noisy background than the system with other parameters. However, we stress that the peak's correct probability in this circumstance is about 0.3, which is much lower than a robust operation required. When the parameters are suitably set ($a=0.5$ and $b=0.15$), typical LSR phenomenon is obtained while the correct probability first increases to 100% and then decreases with the increase of noise intensity.

For OR logic gate, the success probability P as a function of a and b is plotted in Fig. 3, for the three different noise intensities $D=0.3$, 0.5, and 0.7 when $r=0.2$. It shows that the optimal area of a and b shrinks when the noise intensity gets bigger. And smaller value of b and bigger value of a may induce more robust logic operation at high level of noise.

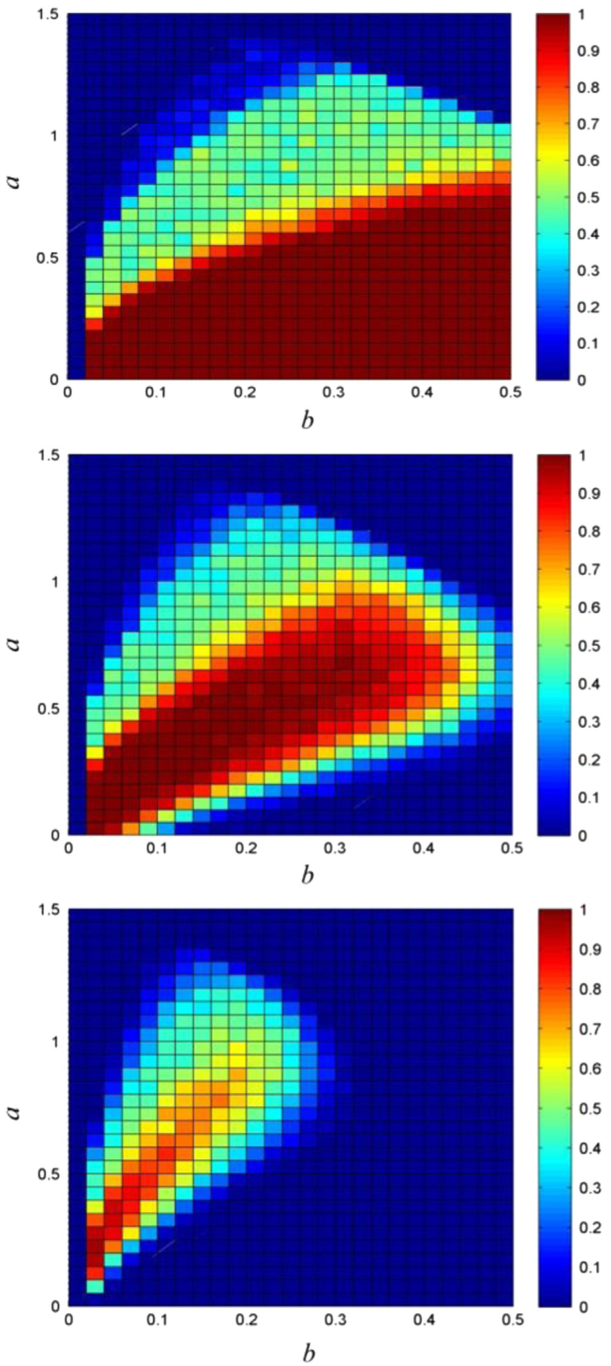

Fig. 3. The density map of the probability P versus a and b is drawn for different noise densities $D=0.3$, 0.5, and 0.7 (from top to bottom), $r=0.2$. It shows that the optimal area of a and b shrinks when the noise intensity gets bigger. And smaller value of b and bigger value of a may induce more robust logic operation at high level of noise.

4. Parameter-induced LSR

In this section, we will focus on the study of how to realize the reliability and flexibility of logical operation through adjusting the parameters of the nonlinear system under a certain noise intensity. We apply the Genetic Algorithm to search for multi-parameters at the same time. It has been proved that modulating multi-parameters synchronously will give better result than searching for the suitable parameter one by one in the conditional SR phenomenon.

We regard the correct probability P of getting desired logic operation as the fitness function. Explicit method of how the Genetic Algorithm is used to achieve the multi-parameters optimization is described as follows:

1) Coding
First of all, searching ranges and accuracy should be set, and then jointly encoded in the binary form. The search interval for a is $[A_{\min}, A_{\max}]$, and that for b is $[B_{\min}, B_{\max}]$. Set the search precision to be δ, so we can determine the corresponding coding length l and k as

$$\begin{cases} 2^l - 1 = (A_{\max} - A_{\min})/\delta \\ 2^k - 1 = (B_{\max} - B_{\min})/\delta \end{cases} \tag{4}$$

The size of chrome is $l+k$, and the corresponding codeword $a_l a_{l-1} a_{l-2} \dots a_1 b_k b_{k-1} b_{k-2} b_{k-3} \dots b_1$ can be obtained. In this work, we set the system parameters a and b to be in the corresponding search ranges [0.01, 1.5] and [0.01, 0.5], and $\delta=0.01$, so $l=8$ and $k=6$, the chrome size is 14.
2) Initializing
Set the population size, select individual randomly, and initialize the group. The size of the population is 20 in this paper.
3) Decoding
The individual coding as: $a_l a_{l-1} a_{l-2,} \dots, a_1 b_k b_{k-1} b_{k-2} b_{k-3,} \dots, b_1$ can be decoded as

$$\begin{cases} a = A_{\min} + \left(\sum_{i=1}^{l} a_i 2^{i-1} \right) \frac{A_{\max} - A_{\min}}{2^l - 1} \\ b = B_{\min} + \left(\sum_{j=1}^{k} b_j 2^{j-1} \right) \frac{B_{\max} - B_{\min}}{2^k - 1} \end{cases} \tag{5}$$

4) Fitness evaluation
In this step, substitute a and b obtained in Eq. (5) to solve Eq. (1) with fourth-order Runge–Kutta algorithm. Then calculate the correct probability P so as to obtain the individual fitness, and rearrange the individual order according to the fitness.
5) Selection, crossover, and mutation
Select superior individuals according to Roulette rule, reserving the individual with best fitness to the next generation, and with a certain probability P_c of crossover (one-point crossover) and P_m mutation (alleles) to produce the next generation. In this work, $P_c=0.8$ and $P_m=0.1$.
6) Termination
Repeat Steps 3–5 until the set iterative condition is met and terminate the cycle. The largest iteration number is 200.

In the case when the bias r is kept as 0.2, the optimal system parameters are searched at different noise intensities, and the result is plotted in Fig. 4. It is evident that multi-parameter optimization can obtain clear logic operation in a wide range of noise densities. It can be seen from Fig. 4 that the amplitude of parameter b decreases smoothly with the increase of D, while a changes greatly and the trend is toward higher value. This consequence is accordant with Fig. 3.

5. Discussion and conclusion

How the changing of a and b affects the behavior of the bistable system can be explained by the potential well depth given by $\Delta U = a^2/4b$. Obviously, increasing a or decreasing b will raise the depth of the potential wells, so the particle needs more power to cross the higher potential barrier, and vice versa. Further, we also find that the depth of the potential well is not the only matter that determines either the system can get the right logic output or not. Experiments

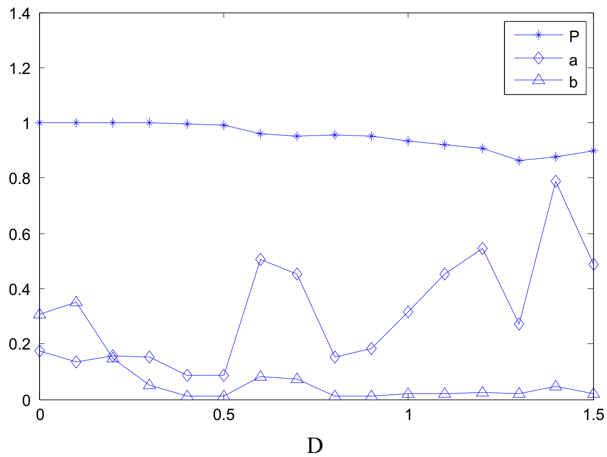

Fig. 4. Vibrations of optimal parameters of a, b and the corresponding correct probability P as a function of D, $r=0.2$.

show that systems with different a and b but same potential well depth respond differently to the same noisy signal. An explanation of this phenomenon is obtained by examining the time taken by the system to cross over the barrier from one well to the other.

In conclusion, parameter-induced LSR in a class of bistable system is studied in this paper. The effects of the parameters of the stochastic system on the response to noisy input are examined, and we apply Genetic Algorithm to search for the best parameters synchronously. Numerical stimulation and experiments show that when the background noise is strong, we should increase a and decrease b to maintain the best state. And the parameter-induced LSR proposed in this work may obtain high robust logic operation in a relatively wide range of noise intensity. Despite LSR has been pointed out to be the intriguing method to help designing large scaled integrated circuits, the range of applicability of LSR is not limited to electronics. In fact it has greater potential in the context of newer paradigms of computing (in particular molecular, chemical or DNA), where the intrinsic noise cannot be eliminated. The characteristic of adjusting the parameters can yield LSR in certain noise intensity which may have practical meaning in these circumstances.

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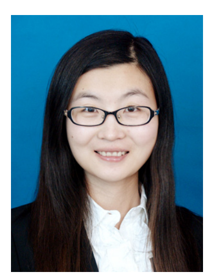

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